NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-785

Disturbing Effects of Attitude Control Maneuvers on the Orbital Motion of the Helios Spacecraft

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

September 15, 1976

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. 33-785	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle DISTURBING EFFECTS OF ATTITUDE CONTROL MANEUVERS ON THE ORBITAL MOTION OF THE HELIOS SPACECRAFT		5. Report Date September 15, 1976 6. Performing Organization Code
7. Author(s) R. M. Georgevic		8. Performing Organization Report No.
9. Performing Organization Name and Address JET PROPULSION LABORATORY California Institute of Technology 4800 Oak Grove Drive Pasadena, California 91103		10. Work Unit No. 11. Contract or Grant No. NAS 7-100 13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS AND SPACE ADMINISTRATION Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The position of the spin axis of the Helios A spacecraft has been maintained and updated by a series of attitude control maneuvers, by means of a sequence of unbalanced jet forces which produce an additional disturbed motion of the spacecraft's center of mass. This report examines the character of this motion, its magnitude and direction. In addition to this, for practical purposes of the orbit determination of the spacecraft, a computer program shows how the components of the disturbing acceleration in the spacecraft-fixed reference frame can be easily computed. The program is given as an appendix to this report.

17. Key Words (Selected by Author(s))	18. Distribution Statement
Spacecraft Communications, Command and Tracking Helios Project	Unclassified -- Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	38	

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-785

Disturbing Effects of Attitude Control Maneuvers on the Orbital Motion of the Helios Spacecraft

R. M. Georgevic

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

September 15, 1976

PREFACE

The work described in this report was performed by the Mission Analysis Division of the Jet Propulsion Laboratory.

CONTENTS

TABLES

FIGURES

ABSTRACT

The position of the spin axis of the Helios A spacecraft has been maintained and updated by a series of attitude control maneuvers, by means of a sequence of unbalanced jet forces which produce an additional disturbed motion of the spacecraft's center of mass. This report examines the character of this motion, its magnitude and direction. In addition to this, for practical purposes of the orbit determination of the spacecraft, a computer program shows how the components of the disturbing acceleration in the spacecraft-fixed reference frame can be easily computed. The program is given as an appendix to this report.

DISTURBING EFFECTS OF ATTITUDE CONTROL MANEUVERS ON THE ORBITAL MOTION OF THE HELIOS SPACECRAFT

R. M. Georgevic

Introduction:

The unbalanced attitude control nozzle firings on the spin-stabilized Helios spacecraft during the sequence of attitude control maneuvers are producing disturbing effects on the orbital motion of the spacecraft. This additional motion of the center of mass of the spacecraft occurs only when the jet force of the nozzle is not counteracted by another jet force of the same magnitude and opposite direction, thus producing a couple which causes a purely rotational motion of the spacecraft.

The jet force is not continuous: it starts when the Sun sensor activates the nozzle and lasts for one quarter of the rotational period of the spacecraft, i.e. for one fourth of a second and then stops, to start again at the beginning of the next second. Each cycle has sixteen such pulses; between each two cycles the nozzle is inactive and produces no force. The maneuvers of the Helios A spacecraft are shown in Table 1.

In further text we shall entirely disregard the rigid-body motions of the spacecraft and examine only the effects of jet forces on its orbital motion, considering the spacecraft as a point-mass. Still, to simplify a rather complicated analysis of the motion under the influence of these forces combined with the motion under other forces acting on the spacecraft, we shall introduce a few restrictions. They are as follows:

1. We shall observe the effects of jet forces during one cycle (16 pulses) separated from the effects of other forces. This can be done because of the short duration of each cycle (16 seconds) and the small magnitude of the jet force (1 Nt).

2. The spinning period of the spacecraft will be assumed constant, e.g. the spin rate will always be 60 rpm.

3. The fundamental plane (xy-plane) of the rotating, spacecraft-fixed system of reference will be the ecliptic plane of 1950.0. This can be assumed because the position of the spin-axis of Helios is almost perfectly colinear with the normal to the ecliptic plane (or at least within tolerable limits).

4. During one cycle of sixteen pulses the spacecraft will be considered stationary, e.g.

$$\bar{r}(t) = \bar{r}(t_o), \ \bar{v}(t) = 0, \ t_o \leq t \leq t_o + 16 \text{ sec.},$$

where t_o is the beginning of the cycle.

5. It will also be assumed that, at the beginning of each cycle, the y-axis (pitch-axis of Helios carrying the Sun sensor) of the rotating spacecraft-fixed reference frame is pointing at the Sun.

Table 1. Helios A spacecraft maneuvers

DAY OF YEAR 1975	HR	MIN	SEC	TYPE	DURATION
41	07	29	00	RP	16 Pulses
	07	38	00	RP	"
	12	06	00	RN	"
	12	15	00	RN	"
	12	24	00	RN	"
	12	33	00	RN	"
	13	56	00	PN	"
	14	24	00	PN	"
	14	52	00	PN	"
	15	20	00	PN	16 Pulses
63	07	55	00	RN	16 Pulses
	08	09	21	RN	"
	08	22	22	RN	"
	08	35	43	RN	"
	09	36	59	PN	"
	10	06	44	PN	"
	14	12	45	PN	"
	14	26	45	SPUP	4 Revolutions
	15	17	39	PN	16 Pulses
	15	49	51	PN	"
	16	15	53	PN	16 Pulses
80	07	29	00	RN	16 Pulses
	07	50	00	RN	"
	08	30	00	RN	"
	08	50	00	RN	"
	09	10	00	RN	"
	09	30	00	RN	"
	09	50	00	RN	16 Pulses
102	09	09	13	PN	16 Pulses
	09	40	00	PN	"
	09	50	00	PN	"
	10	00	00	PN	"
	10	10	00	PN	"
	10	20	00	PN	"
	10	30	00	PN	"
	10	50	00	RN	"
	11	00	00	RN	"
	11	10	00	RN	"
	11	20	00	RN	16 Pulses

RP = ROLL POSITIVE

RN = ROLL NEGATIVE

PN = PITCH NEGATIVE

SPUP = SPIN UP.

The Three Reference Frames

Without specifying the positions of their origins, we shall introduce two ecliptic reference systems: the first one, XYZ is ecliptic, space-fixed system, with its X-axis toward the vernal equinox point; the second is the system rotating with the spacecraft at 60 rpm. Both systems are shown on Figure 1. The relationship between the two systems is given by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} , \qquad (1)$$

where ϕ is the angle shown on Figure 1. It is obvious that, due to the spacecraft's rotation, the angle ϕ is proportional to ωt.

Introducing now the space-fixed equatorial frame of reference, X_Q, Y_Q, Z_Q, we find that

$$\begin{pmatrix} X_Q \\ Y_Q \\ Z_Q \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} , \qquad (2)$$

and, combining (1) and (2), we obtain

$$\begin{pmatrix} X_Q \\ Y_Q \\ Z_Q \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} ,$$

or

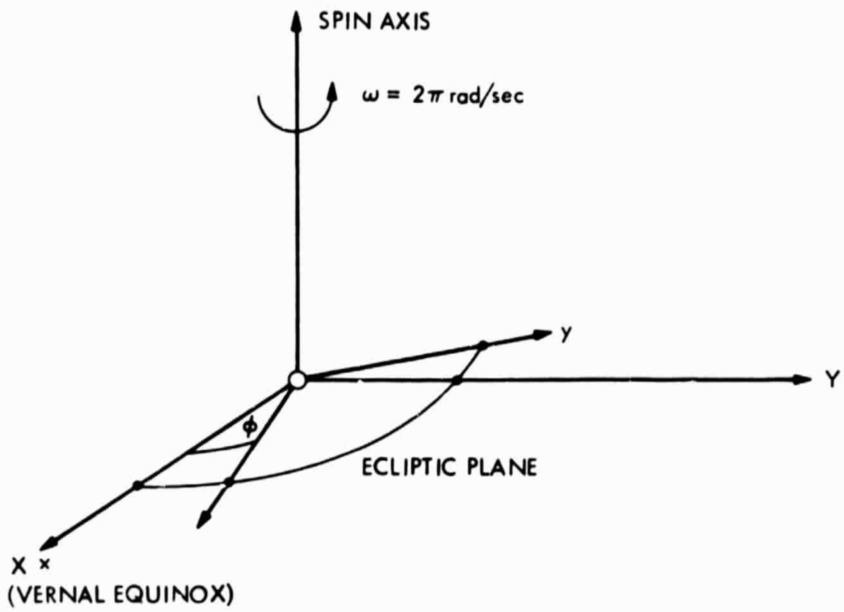

Fig. 1. Ecliptic reference systems

$$\begin{pmatrix} X_Q \\ Y_Q \\ Z_Q \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \cos\varepsilon \sin\phi & \cos\varepsilon \cos\phi & -\sin\varepsilon \\ \sin\varepsilon \sin\phi & \sin\varepsilon \cos\phi & \cos\varepsilon \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad . \tag{3}$$

The Position of the Attitude Control Valve

The pitch and roll motions of Helios are determined with respect to another reference frame which, for the time of duration of one maneuvering cycle, can be considered stationary (x_1 y_1 z_1 - system). The x_1y_1-plane of this system coincides with the ecliptic plane and the y_1-axis (pitch axis) is always in the spacecraft-sun direction (Figure 2). The angle λ_s is the longitude of the Sun observed from the spacecraft, which is

$$\lambda_s = 180^o + \lambda,$$

where λ is the heliocentric longitude of the spacecraft. Hence,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\sin\lambda & -\cos\lambda & 0 \\ \cos\lambda & -\sin\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \tag{4}$$

Combining Equations (i) and (4) we find

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin(\phi-\lambda) & -\cos(\phi-\lambda) & 0 \\ \cos(\phi-\lambda) & \sin(\phi-\lambda) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \tag{5}$$

Measuring time since the moment when the two systems coincide, we obtain

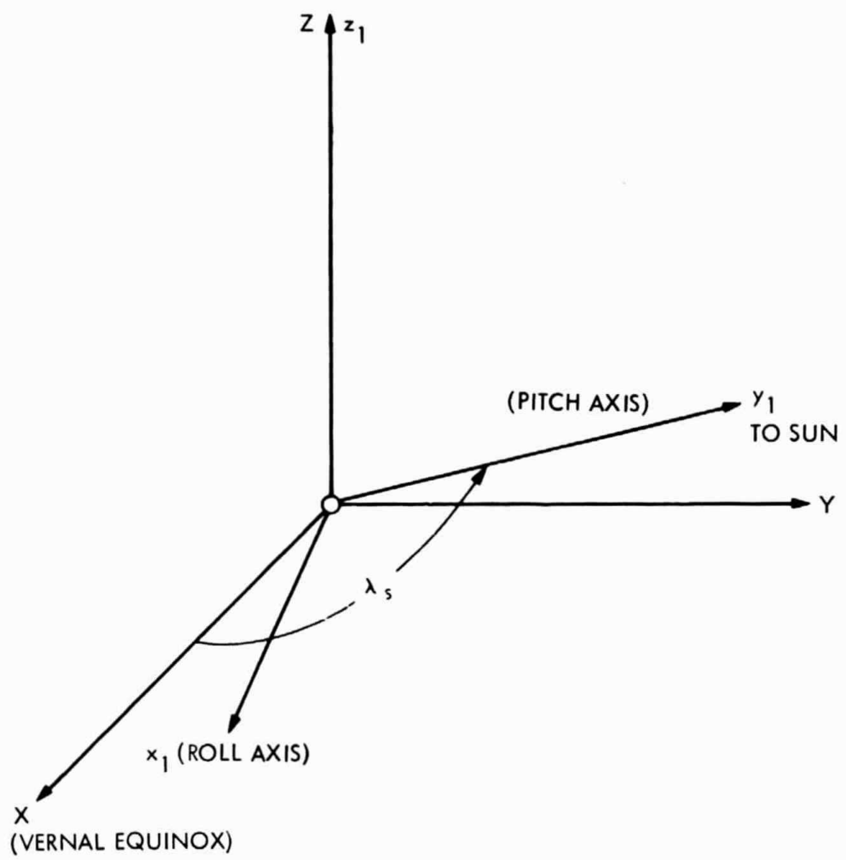

Fig. 2. Spacecraft body-fixed pitch-yaw-roll system

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \omega t & \sin \omega t & 0 \\ -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} . \tag{6}$$

Comparison of Equations (5) and (6) yields

$$\phi = \lambda + \omega t + \frac{\pi}{2} , \tag{7}$$

and, for $t = 0$ $(y \equiv y_1)$,

$$\phi_o = \lambda + \frac{\pi}{2} .$$

The position of the precession valve is shown on Figure 3. The angle θ is approximately 38^o. The nozzle is located on the lowest brim of the truncated cone of the Helios body. The line drawn from the center of mass of the spacecraft (C.M. on Figure 3) to the nozzle is perpendicular to the axis of symmetry of the nozzle - the line of action of the jet force. The nozzle lies in a plane parallel to the yz-plane of the body-fixed rotating reference frame. The magnitude of the jet force is

$$F = |\dot{m}| v_e \cong 1 \text{ Nt},$$

where $\dot{m}$ is the gas flow and v_e is the nozzle velocity of the expelled gas. The magnitude of the acceleration of this force is (mass of the spacecraft is m = 356.9 kg):

$$a = 2.802 \times 10^{-6} \text{ km/sec}^2.$$

To estimate the influence of only one nozzle firing in the duration of one fourth of a second, on the motion of the center of mass of the spacecraft, we find first the change in velocity after the firing, which is

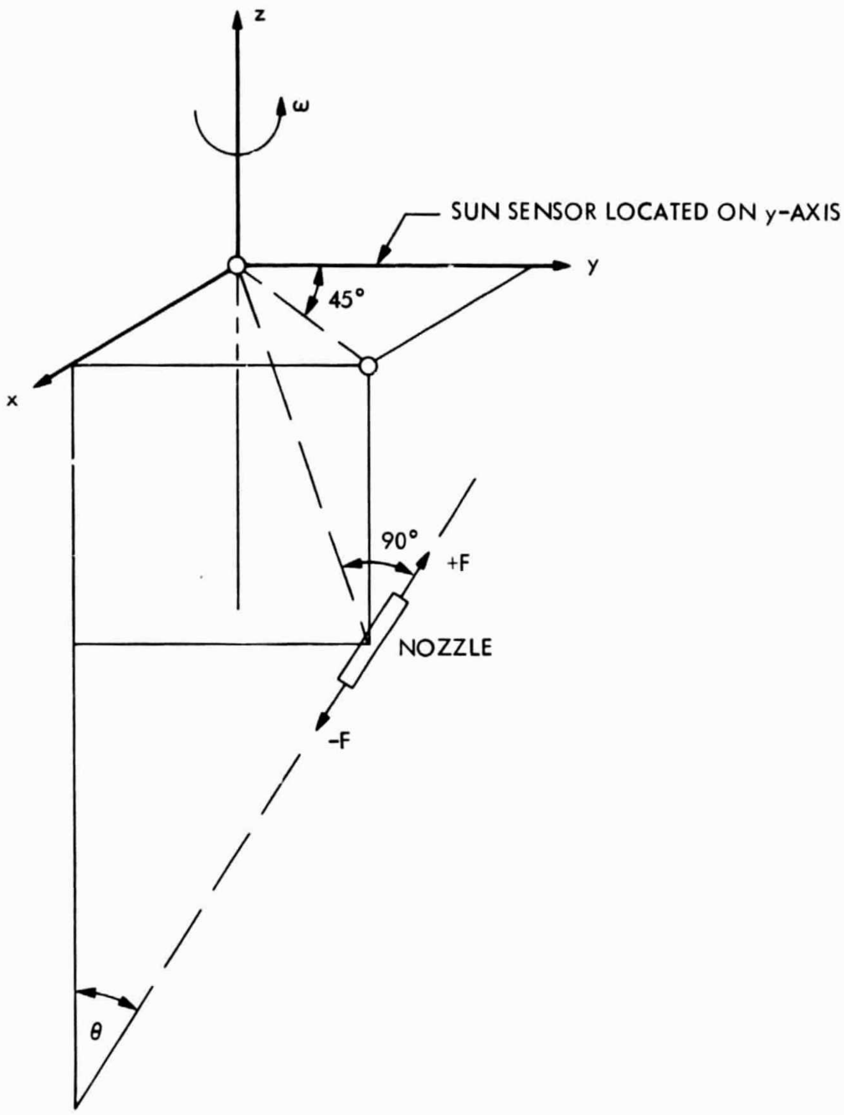

Fig 3. Geometry of the maneuvers

$$\Delta v = 7.0 \times 10^{-7} \text{ km/sec.}$$

Disregarding the orbital motion of the spacecraft, this change in velocity produces a change in the spacecraft's position in an amount of

$$|\Delta \bar{r}| = 60.5 \text{ m/day.}$$

Types of Maneuvers

From Table 1. we see that there are four different types of maneuvers; they are:

RP - Positive roll rotation abouts the +x-axis

RN - Negative roll rotation about the +x-axis.

PN - Negative pitch rotation about the +y-axis

SPUP - (Spin up) Positive rotation about the +z-axis: increase of the spin rate.

The last type of maneuver does not produce any motion of the center of mass of the spacecraft. The rotation rate is changed by a couple of jet valves which operate in a plane perpendicular to the spin-axis and generate a couple; therefore, assuming that the jet forces on both exhaust valves are balanced (or close to being equal), there is no excess force acting on the center of mass.

To create a rotational motion purely about the x_1-axis (roll motion), the precession valve is activated when the angle between the y-axis and the y_1-axis (Sun direction) is 45^o (actually -45°). The jet force creates also a pitch motion but, since the valve operates only for a quarter of a second (one quadrant), during the first one-eighth of a second the pitch motion is negative and during the next one-eighth of a second it is positive and same in magnitude. Hence the total pitch motion over the complete firing period is zero and the outcome is a purely roll motion. The same is done to produce a pure rotation about the y_1-axis, by cancelling out the roll motion.

Since we are not concerned with rotational motions of the spacecraft, we need to know only in which direction the jet force is acting for each particular type of maneuver. From Figure 3 we can see that the PN and RP motions are produced by a +F force (out of ecliptic plane to the northern hemisphere), and that RN motion is produced by a -F force (out of the ecliptic plane, south).

RP, RN and PN motions are schematically shown on Figure 4, and listed in Table 2.

Components of the Force in the Equatorial, Space-Fixed Reference Frame

From Figure 3 we see that the components of the jet force, along the axes of the rotating, body-fixed reference frame are

$$\begin{pmatrix} 0 \\ f \sin \theta \\ f \cos \theta \end{pmatrix},$$

where

$$f = \begin{cases} +F \text{ for RP and PN maneuvers} \\ -F \text{ for RN maneuvers} \end{cases}$$

The components of the force in the ecliptic, space-fixed reference frame are

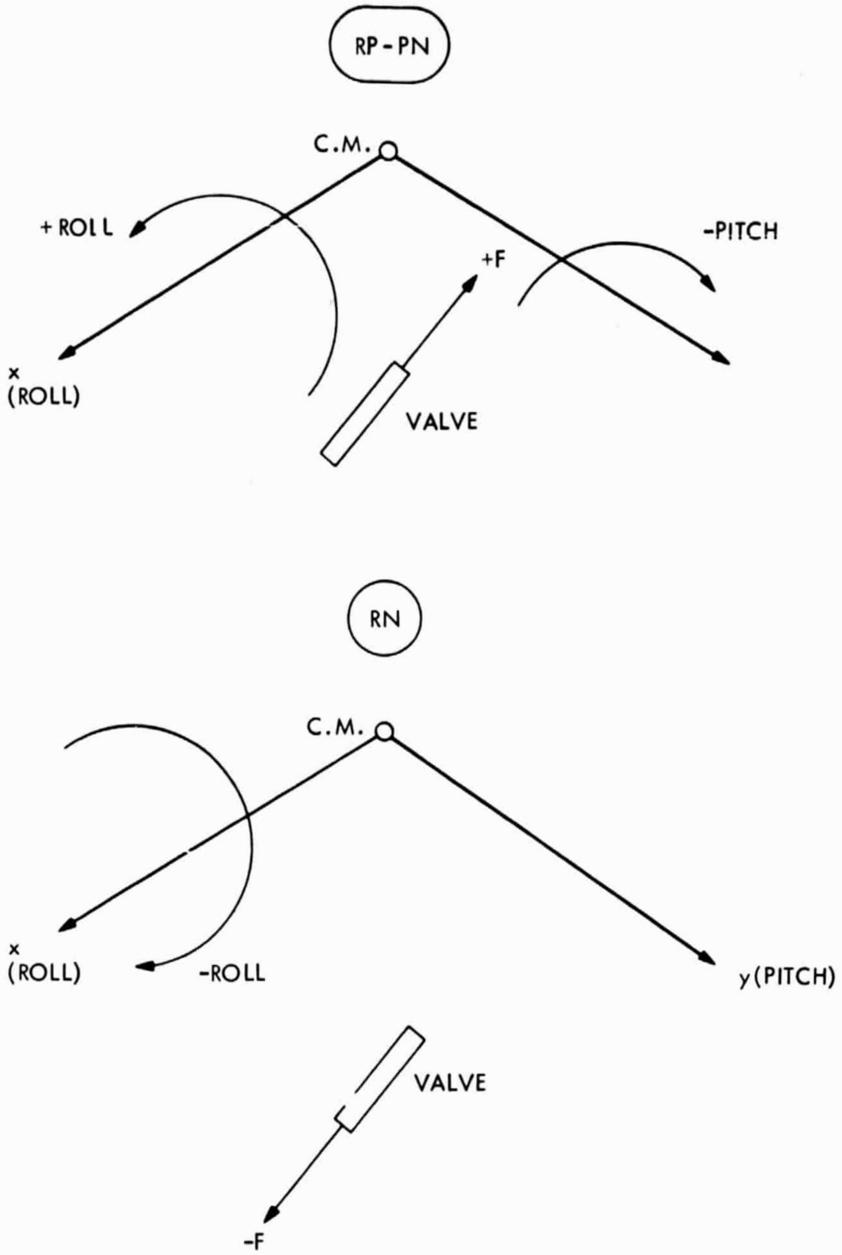

Fig. 4. Schematic view of the maneuvers

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ f \sin\theta \\ f \cos\theta \end{pmatrix}$$

and in the space-fixed equatorial reference frame,

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \cos\varepsilon \sin\phi & \cos\varepsilon \cos\phi & -\sin\varepsilon \\ \sin\varepsilon \sin\phi & \sin\varepsilon \cos\phi & \cos\varepsilon \end{pmatrix} \begin{pmatrix} 0 \\ f \cos\theta \\ f \cos\theta \end{pmatrix} ,$$

or

$$\left.\begin{aligned} F_x &= -f \sin\theta \sin\phi, \\ F_y &= f(\cos\varepsilon \sin\theta \cos\phi - \sin\varepsilon \cos\theta), \\ F_z &= f(\sin\varepsilon \sin\theta \cos\phi + \cos\varepsilon \cos\theta). \end{aligned}\right\} \qquad (8)$$

where the angle ϕ is given explicitly in Equation (7).

Table 2. Types of maneuvers

Maneuver Type	Means	Force
RP	Positive Roll Motion	+F
RN	Negative Roll Motion	-F
PN	Negative Pitch Motion	+F

Integration of Equations of Motion

Since the angular orbital motion of the spacecraft, at the rate of $\dot{\lambda} = 3.8 \times 10^{-7}$ rad/sec is negligible in comparison with the rotation rate $\omega = 2\pi$ rad/sec, we shall consider λ as constant. Equations of motion are

$$\begin{pmatrix} \ddot{X}_Q \\ \ddot{Y}_Q \\ \ddot{Z}_Q \end{pmatrix} = \frac{f}{m} \begin{pmatrix} - \sin\theta \sin\phi \\ \cos\varepsilon \sin\theta \cos\phi - \sin\varepsilon \cos\theta \\ \sin\varepsilon \sin\theta \cos\phi + \cos\varepsilon \cos\theta \end{pmatrix},$$

where $\phi = \lambda + \omega t + \frac{\pi}{2}$. At the beginning of the first valve firing $t = 0$ and, at the end of the first pulse $\omega t = \frac{\pi}{2}$. Also, $d\phi = \omega dt$. The first integration yields ($\phi_o = \lambda + \frac{\pi}{2}$) the change of the velocity of the center of mass of the spacecraft due to the action of the jet force in the form

$$\begin{pmatrix} \Delta\dot{X}_Q \\ \Delta\dot{Y}_Q \\ \Delta\dot{Z}_Q \end{pmatrix} = \frac{f}{m\omega} \begin{pmatrix} -(\cos\phi - \cos\phi_o)\sin\theta \\ \cos\varepsilon(\sin\phi - \sin\phi_o)\sin\theta - (\phi-\phi_o)\sin\varepsilon \cos\theta \\ \sin\varepsilon(\sin\phi - \sin\phi_o)\sin\theta + (\phi-\phi_o)\cos\varepsilon \cos\theta \end{pmatrix},$$

where $\phi_o \le \phi \le \frac{\pi}{2}$, and $\phi_o = \lambda + \frac{\pi}{2}$. Finally, we find

$$\begin{pmatrix} \Delta\dot{X}_Q \\ \Delta\dot{Y}_Q \\ \Delta\dot{Z}_Q \end{pmatrix} = \frac{f}{m\omega} \begin{pmatrix} -(\cos\phi + \sin\lambda)\sin\theta \\ \cos\varepsilon(\sin\phi - \cos\lambda)\sin\theta - \omega t \sin\varepsilon \cos\theta \\ \sin\varepsilon(\sin\phi - \cos\lambda)\sin\theta + \omega t \cos\varepsilon \cos\theta \end{pmatrix}. \tag{9}$$

At the end of the first pulse, the change in the spacecraft's velocity is ($\phi = \pi + \lambda$):

$$\Delta\bar{v} = \frac{f}{m\omega}\begin{pmatrix} -(\sin\lambda - \cos\lambda)\sin\theta \\ -(\sin\lambda + \cos\lambda)\cos\varepsilon\sin\theta - \frac{\pi}{2}\sin\varepsilon\cos\theta \\ -(\sin\lambda + \cos\lambda)\sin\varepsilon\sin\theta + \frac{\pi}{2}\cos\varepsilon\cos\theta \end{pmatrix}. \tag{10}$$

Integrating Equations (9) once again we obtain

$$\begin{pmatrix} \Delta X_Q \\ \Delta Y_Q \\ \Delta Z_Q \end{pmatrix} = -\frac{f}{m\omega^2}\begin{pmatrix} (\sin\phi - \cos\lambda + \omega t\sin\lambda)\sin\theta \\ (\cos\phi + \sin\lambda + \omega t\cos\lambda)\cos\varepsilon\sin\theta + \frac{(\omega t)^2}{2}\sin\varepsilon\cos\theta \\ (\cos\phi + \sin\lambda + \omega t\cos\lambda)\sin\varepsilon\sin\theta - \frac{(\omega t)^2}{2}\cos\varepsilon\cos\theta \end{pmatrix}. \tag{11}$$

At the end of the first pulse the change in the spacecraft's position is

$$\Delta\bar{r} = -\frac{f}{m\omega^2}\begin{pmatrix} (-\sin\lambda - \cos\lambda + \frac{\pi}{2}\sin\lambda)\sin\theta \\ (-\cos\lambda + \sin\lambda + \frac{\pi}{2}\cos\lambda)\cos\varepsilon\sin\theta + \frac{\pi^2}{8}\sin\varepsilon\cos\theta \\ (-\cos\lambda + \sin\lambda + \frac{\pi}{2}\cos\lambda)\sin\varepsilon\sin\theta - \frac{\pi^2}{8}\cos\varepsilon\cos\theta \end{pmatrix}. \tag{12}$$

Geometry of Motion

In order to interpret the motion of the center of mass, described by Equations (11), we shall assume that, without any loss of generality, $X_Q(0) = Y_Q(0) = Z_Q(0) = 0$, and $\lambda = -\frac{\pi}{2}$. Then $\phi = \omega t$, and Equations (11) yield

$$\begin{pmatrix} X_Q \\ \\ Y_Q \\ \\ Z_Q \end{pmatrix} = \frac{f}{m\omega^2} \begin{pmatrix} (\phi - \sin\phi)\ \sin\theta \\ \\ (1 - \cos\phi)\ \cos\varepsilon\ \sin\theta - \frac{\phi^2}{2} \sin\varepsilon\ \cos\theta \\ \\ (1 - \cos\phi)\ \sin\varepsilon\ \sin\theta + \frac{\phi^2}{2}\ \cos\varepsilon\ \cos\theta \end{pmatrix}$$

In the space-fixed, ecliptic coordinates this motion is expressed by

$$\begin{cases} X = k(\phi - \sin\phi) \quad , \\ Y = k(1 - \cos\phi) \quad , \\ Z = k_1\ \phi^2 \quad , \end{cases}$$

where $\phi = \omega t$, and

$$k = \frac{f \sin\theta}{m\omega^2}, \qquad k_1 = \frac{f \cos\theta}{2m\omega^2} \quad .$$

It is obvious that the center of mass moves on a cycloidal cylinder (looking more like wavy roofing material), parallel to the Z-axis, and proportionally to t^2 (or ϕ^2) up or down the surface of the cylinder, depending on whether f is positive or negative. The radius of the generating circle of the cycloid is

$$k = \frac{f \sin\theta}{m\omega^2} \quad .$$

The rate at which the center of mass moves in Z-direction (out of the ecliptic plane) is

$$\frac{f \cos\theta}{m} t \quad ,$$

and it is obvious that for $\theta = 90°$ the center of mass remains in the ecliptic plane. With $\theta = 38°$, m = 356.9 kg, and f = 1 Nt, this rate is given in km/sec by

$$2.2 \times 10^{-6} \; t,$$

where t is in seconds of time. This means that, due to the action of only one pulse, the spacecraft will move 47.5 m out of the ecliptic plane in one day.

The Motion During One Cycle (16 Pulses)

It is obvious that, during one complete cycle of sixteen pulses, the disturbing effects will accumulate. For instance, if $\Delta\bar{r}$ and $\Delta\bar{v}$ [given respectively by Equations (12) and (10)] are respective changes in the position of the spacecraft and its velocity at the end of the first pulse lasting one quarter of a second, the total change in $\bar{r}$ and $\bar{v}$ after one second will be

$$\Delta\bar{r}_1 = \Delta\bar{r} + \frac{3}{4}\Delta\bar{v}$$

$$\Delta\bar{v}_1 = \Delta\bar{v} \quad .$$

At the end of the second firing (second cycle) the total changes in $\bar{r}$ and $\bar{v}$ will be

$$\Delta\bar{r}_1 + \Delta\bar{r} + \frac{1}{4}\Delta\bar{v} = 2\Delta\bar{r} + \Delta\bar{v}$$

$$\Delta\bar{v} + \Delta\bar{v} = 2\Delta\bar{v} \quad ,$$

because the effects of the jet force during each pulse are the same if it is the same type of maneuver. After two seconds these changes will be

$$\Delta\bar{r}_2 = 2\Delta\bar{r} + \Delta\bar{v} + \frac{3}{4}\Delta\bar{v} = 2\Delta\bar{r} + (1 + 2\frac{3}{4})\,\Delta\bar{v}$$

$$\Delta\bar{v}_2 = 2\Delta\bar{v}$$

etc. For instance,

$$\Delta\bar{r}_3 = 3\Delta\bar{r} + (1 + 2 + 3.\frac{3}{4})\,\Delta\bar{v}$$

$$\Delta\bar{v}_3 = 3\Delta\bar{v} \quad ,$$

$$\Delta\bar{r}_4 = 4\Delta\bar{r} + (1 + 2 \qquad \Delta\bar{v} \quad ,$$

$$\Delta\bar{v}_4 = 4\Delta\bar{v} \quad .$$

At the end of the n-th second the changes in $\bar{r}$ and $\bar{v}$ are

$$\Delta\bar{r}_n = n\Delta\bar{r} + \frac{n(2n + 1)}{4}\Delta\bar{v} \tag{13}$$

$$\Delta\bar{v}_n = n\Delta\bar{v} \tag{14}$$

where $\Delta\bar{r}$ and $\Delta\bar{v}$ are given by Equations (12 and (10), respectively. For n = 16 (end of one cycle)

$$\begin{aligned} \Delta\bar{r}_c &= 16\ \Delta\bar{r} + 132\ \Delta\bar{v} \\ \Delta\bar{v}_c &= 16\ \Delta\bar{v}\ . \end{aligned} \tag{15}$$

One Example

To illustrate the solution and to give a physical interpretation of the effects of the maneuvers on the position and velocity of the spacecraft's center of mass, we shall calculate the changes $\Delta\bar{r}_c$ and $\Delta\bar{v}_c$ after one complete cycle of firings. To simplify the calculations we shall assume that $\lambda = 0$ and obtain $\Delta\bar{r}$ and $\Delta\bar{v}$ respectively from Equations (12) and (10) (for the obliquity of the ecliptic we shall take the value $\varepsilon = 23^\circ\!.4458$). We find

$$\Delta\bar{r} = -\frac{f}{m\omega^2} \begin{pmatrix} -\sin\theta \\ (\frac{\pi}{2} - 1)\cos\varepsilon\ \sin\theta + \frac{\pi^2}{8}\sin\varepsilon\ \cos\theta \\ (\frac{\pi}{2} - 1)\sin\varepsilon\ \sin\theta - \frac{\pi^2}{8}\cos\varepsilon\ \cos\theta \end{pmatrix}$$

or

$$\Delta\bar{r} = \begin{pmatrix} 4.3695 \\ -5.0335 \\ 5.3378 \end{pmatrix} \times 10^{-8}\ \mathrm{km}$$

and

$$\Delta\bar{v} = \frac{f}{m\omega}\begin{pmatrix} \sin\theta \\ -\cos\varepsilon\ \sin\theta - \frac{\pi}{2}\ \sin\varepsilon\ \cos\theta \\ -\sin\varepsilon\ \sin\theta + \frac{\pi}{2}\ \cos\varepsilon\ \cos\theta \end{pmatrix},$$

or

$$\Delta\bar{v} = \begin{pmatrix} 2.7455 \\ -4.7150 \\ 3.9717 \end{pmatrix} \times 10^{-7}\ \text{km/sec.}$$

Hence,

$$\Delta\bar{r}_c = \begin{pmatrix} 3.6940 \\ -6.3043 \\ 5.3280 \end{pmatrix} \times 10^{-5}\ \text{km}$$

$$\Delta\bar{v}_c = \begin{pmatrix} 4.3928 \\ -7.5440 \\ 6.3547 \end{pmatrix} \times 10^{-6}\ \text{km/sec}$$

and,

$$|\Delta\bar{r}| = 9.043 \times 10^{-2}\ \text{m}, \qquad |\Delta\bar{v}| = 1.080 \times 10^{-2}\ \text{m/sec.}$$

The final velocity at the end of only one cycle would produce a change in the spacecraft's position of 0.93 km in one day.

The Computer Program

A computer program, based on an unperturbed elliptical orbit of Helios, is written in such a manner that it yields the heliocentric ecliptic longitude of the spacecraft, λ, which appears in Equations (10) and (12). The program computes the accumulated effects of the disturbing force in the heliocentric position and velocity of the spacecraft during first four maneuvers and can be extended to as many maneuvers as desirable. The cumulative effects on Helios A during the four maneuvers listed in Table 1 are given in Table 3. The complete listing of the program is given in the Appendix.

The Implementation of the Perturbative Acceleration in the Orbit Determination Program

The disturbing effects in the position and velocity of the spacecraft, given by Equations (15), cannot be implemented into the Orbit Determination Program, therefore, the accelerations, producing these effects, should be included directly in the input data block of the program. The step-function accelerations, given by the three equations following Equations (8) cannot be introduced into the program either, as the program does not provide a mathematical model for such accelerations.

There is, however, no constant acceleration, $\bar{a}$, which, at the end of one cycle of 16 seconds, would produce changes $\Delta\bar{r}_c$ and $\Delta\bar{v}_c$, specified by Equations (15). If there was such an acceleration, then the equations

$$\Delta\bar{r}_c = \frac{1}{2}\bar{a}T^2 ,$$

Table 3. Accumulated disturbing effects in coordinates during maneuvers

MANOEUVRE NO. 1

DX =	-27.36286996	KM
DY =	-22.43863869	KM
DZ =	34.05021238	KM
DVX =	-.00001210	KM/SEC
DVY =	-.00000961	KM/SEC
DVZ =	.00001508	KM/SEC

MANOEUVRE NO. 2

DX =	10.78901064	KM
DY =	2.24149281	KM
DZ =	-17.35809875	KM
DVX =	-.00001119	KM/SEC
DVY =	-.00000206	KM/SEC
DVZ =	.00001836	KM/SEC

MANOEUVRE NO. 3

DX =	-.32380591	KM
DY =	-3.15407485	KM
DZ =	-15.28499794	KM
DVX =	-.00000183	KM/SEC
DVY =	-.00001526	KM/SEC
DVZ =	-.00007400	KM/SEC

MANOEUVRE NO. 4

DX =	9.69688976	KM
DY =	-12.93451202	KM
DZ =	12.74077559	KM
DVX =	.00001528	KM/SEC
DVY =	-.00002034	KM/SEC
DVZ =	.00002006	KM/SEC

and

$$\Delta\bar{v}_c = \bar{a}T$$

with $\Delta\bar{r}_c$ and $\Delta\bar{v}_c$ given, and T = 16 sec, should be satisfied simultaneously. This, of course, is impossible, except in the case when $2\Delta\bar{r}_c = T\Delta\bar{v}_c$, or $\Delta\bar{r}_c = 8\Delta\bar{v}_c$, which is out of the question.

Since the orbit determination program provides an option for occasional accelerations, the problem can be solved in the following manner. Let $\bar{a}_1$ be the acceleration given to the center of mass of the spacecraft at the beginning of a certain cycle, and $\bar{a}_2$ be the acceleration added in the middle of the cycle, i.e. eight seconds after the first acceleration, $\bar{a}_1$, has started acting on the body; both $\bar{a}_1$ and $\bar{a}_2$ should be constant vectors and their combined actions should produce exactly the changes in the position and velocity of the spacecraft, $\Delta\bar{r}_c$ and $\Delta\bar{v}_c$ respectively, at the end of the cycle.

Equations describing this dynamic event are

$$\frac{1}{2}\,\bar{a}_1T^2 + \frac{1}{2}\,\bar{a}_2\,\left(\frac{T}{2}\right)^2 = \Delta\bar{r}_c \quad ,$$

and

$$\bar{a}_1T + \bar{a}_2\,\left(\frac{T}{2}\right) = \Delta\bar{v}_c \quad ,$$

where T = 16 sec., or

$$\left.\begin{aligned} 128\,\bar{a}_1 + 32\,\bar{a}_2 &= \Delta\bar{r}_c \quad , \\ 16\,\bar{a}_1 + 8\,\bar{a}_2 &= \Delta\bar{v}_c \quad . \end{aligned}\right\}$$

Solving for $\bar{a}_1$ and $\bar{a}_2$ we find

$$\bar{a}_1 = \frac{1}{64} (\Delta\bar{r}_c - 4\Delta\bar{v}_c) \quad ,$$
$$\bar{a}_2 = -\frac{1}{32} (\Delta\bar{r}_c - 8\Delta\bar{v}_c) \quad . \tag{16}$$

Expressed in terms of changes $\Delta\bar{r}$ and $\Delta\bar{v}$, respectively given by Equations (12) and (10), $\bar{a}_1$ and $\bar{a}_2$ become

$$\bar{a}_1 = 0.25\ \Delta\bar{r} + 1.0625\ \Delta\bar{v} \quad ,$$
$$\bar{a}_2 = -\ 0.5\ \Delta\bar{r} - 0.125\ \Delta\bar{v}$$

It is evident that $\bar{a}_1$ and $\bar{a}_2$ can be introduced arbitrarily at any time t_1 and t_2 such that

$$0 \leq t_1 \leq t_2 \leq T = 16 \text{ sec.}$$

In that case, the two starting equations would have the forms

$$\frac{1}{2}\bar{a}_1 \tau_1^{\,2} + \frac{1}{2}\bar{a}_2 \tau_2^{\,2} = \Delta\bar{r}_c \quad ,$$

$$\bar{a}_1 \tau_1 + \bar{a}_2 \tau_2 = \Delta\bar{v}_c \quad ,$$

where $\tau_1 = T - t_1$, $\tau_2 = T - t_2$. From these equations we obtain $(\tau_1 \neq \tau_2)$

$$\bar{a}_1 = \frac{2}{\tau_1 (\tau_1 - \tau_2)} \left(\Delta\bar{r}_c - \frac{\tau_2}{2}\Delta\bar{v}_c\right) \quad ,$$

$$\bar{a}_2 = \frac{2}{\tau_2 (\tau_1 - \tau_2)} \left(\Delta\bar{r}_c - \frac{\tau_1}{2}\Delta\bar{v}_c\right) \quad .$$

Accelerations In The Sun-Canopus Oriented System

The system of reference axes, used in the Orbit Determination Program, is generated by means of the spacecraft-Sun and the spacecraft-star Canopus directions. This is, of course, a rotating, non-inertial system, since the spacecraft moves in its orbit around the Sun. The spacecraft-Canopus direction, for all practical purposes, can be considered space-fixed.

The three unit vectors along the axes of the system are given by (Reference 1)

$$\bar{e}_1 = \frac{(\bar{U}_c \times \bar{r}) \times \bar{r}}{r \, |\bar{U}_c \times \bar{r}|} ,$$

$$\bar{e}_2 = \frac{\bar{U}_c \times \bar{r}}{|\bar{U}_c \times \bar{r}|} ,$$

$$\bar{e}_3 = \frac{\bar{r}}{r} ,$$

where

$$\bar{U}_c = \begin{pmatrix} \cos\alpha \cos\delta \\ \sin\alpha \cos\delta \\ \sin\delta \end{pmatrix}$$

is the unit vector along the spacecraft-Canopus direction. α and δ are respectively the right ascension and declination of Canopus for 1950.0 and their respective values are

$$\alpha = 98\overset{\circ}{.}02255$$

$$\delta = -\,68\overset{\circ}{.}98877 .$$

The components of accelerations $\bar{a}_i$, $i = 1, 2$ in this system of reference are then given by

$$\bar{a}_i = \begin{pmatrix} \bar{a}_i \cdot \bar{e}_1 \\ \bar{a}_i \cdot \bar{e}_2 \\ \bar{a}_i \cdot \bar{e}_3 \end{pmatrix} \quad ;$$

their values for the first four maneuvers of the Helios A spacecraft, obtained from the previously mentioned computer program, are listed in Table 4.

Table 4. Helios attitude control maneuvers, disturbing accelerations

TIME	XACC	YACC	ZACC
52.31180525	.575255093-06	-.447974720-06	.115085824-06
52.31189775	-.985046418-07	.704783474-07	-.139059737-07
52.31805515	.575554921-06	-.443400040-06	.130297547-06
52.31814766	-.985416406-07	.779102773-07	-.165808558-07
52.50416660	-.575610756-06	.442227012-06	-.133985235-06
52.50425911	.985476305-07	-.777615581-07	.172306311-07
52.51041651	-.575612482-06	.442189908-06	-.134100214-06
52.51050901	.985478206-07	-.777568401-07	.172509103-07
52.51666641	-.575614010-06	.442155951-06	-.134205596-06
52.51675892	.985479938-07	-.777525013-07	.172694923-07
52.52291632	-.575615545-06	.442122008-06	-.134310850-06
52.52300882	.985481474-07	-.777481901-07	.172880525-07
52.58055544	.575629727-06	-.441807515-06	.135281400-06
52.58064795	-.985496236-07	.777080542-07	-.174591661-07
52.59999990	.575634573-06	-.441700799-06	.135608921-06
52.60009241	-.985501449-07	.776944020-07	-.175169310-07
52.61944437	.575639426-06	-.441593770-06	.135936469-06
52.61953688	-.985506441-07	.776807152-07	-.175746884-07
52.63888884	.575644314-06	-.441486392-06	.136264097-06
52.63898134	-.985511663-07	.776669475-07	-.176324702-07
74.32986067	-.602996828-06	.277100138-07	-.424819373-06
74.32995319	.102998788-06	-.107984350-07	.730071035-07
74.33982563	-.603019330-06	.273417946-07	-.424811287-06
74.33991814	.103002734-06	-.107351568-07	.730108711-07
74.34886551	-.603039759-06	.270075677-07	-.424803687-06
74.34895802	.103006315-06	-.106777183-07	.730142435-07
74.35813610	-.603060677-06	.266645799-07	-.424795612-06
74.35822868	.103009971-06	-.106187721-07	.730176604-07
74.40068245	.603156799-06	-.250888839-07	.424755154-06
74.40077496	-.103026809-06	.103479314-07	-.730328003-07
74.42134190	.603203482-06	-.243226767-07	.424733425-06
74.42143440	-.103034999-06	.102161923-07	-.730397938-07
74.59218693	.603589662-06	-.179575945-07	.424501547-06
74.59227943	-.103102759-06	.912110032-08	-.730891339-07
74.63725662	.603691561-06	-.162700431-07	.424424655-06
74.63734913	-.103120626-06	.883055407-08	-.730995575-07
74.65961742	.603742130-06	-.154314943-07	.424384048-06
74.65970993	-.103129521-06	.868613892-08	-.731043386-07
74.67769623	.603783029-06	-.147528176-07	.424350006-06
74.67778873	-.103136708-06	.856924265-08	-.731079934-07

Table 4. (contd)

TIME	XACC	YACC	ZACC
91.31180477	-.601555598-06	-.271563319-06	.330497748-06
91.31189728	.103082892-06	.513031675-07	-.528894981-07
91.32638836	-.601534701-06	-.270527792-06	.331383823-06
91.32648087	.103078673-06	.511381604-07	-.530572310-07
91.35416603	-.601495628-06	-.268545048-06	.333063184-06
91.35425854	.103070738-06	.508219724-07	-.533754330-07
91.36805534	-.601476444-06	-.267548671-06	.333896658-06
91.36814785	.103066859-06	.506629672-07	-.535338915-07
91.38194370	-.601457515-06	-.266549097-06	.334731187-06
91.38203621	.103063010-06	.505033713-07	-.536918918-07
91.39583302	-.601438792-06	-.265545978-06	.335561047-06
91.39592552	.103059187-06	.503431249-07	-.538494787-07
91.40972137	-.601420354-06	-.264539771-06	.336387885-06
91.40981388	.103055419-06	.501823059-07	-.540065939-07
113.38140011	.593716429-06	.203435000-06	.388535110-06
113.38149261	-.102081576-06	-.293520523-07	-.690883590-07
113.40277767	.593680618-06	.204231828-06	.388171621-06
113.40287018	-.102074916-06	-.294939100-07	-.690377711-07
113.40972137	.593669014-06	.204490313-06	.388053248-06
113.40981388	-.102072748-06	-.295399296-07	-.690212891-07
113.41666603	.593657447-06	.204748542-06	.387934794-06
113.41675854	-.102070597-06	-.295859222-07	-.690047877-07
113.42361069	.593645886-06	.205006710-06	.387816105-06
113.42370319	-.102068436-06	-.296318827-07	-.689882400-07
113.43055534	.593634340-06	.205264602-06	.387697344-06
113.43064785	-.102066296-06	-.296778229-07	-.689716764-07
113.43750000	.593622829-06	.205522344-06	.387578428-06
113.43759251	-.102064151-06	-.297237217-07	-.689550985-07
113.45138836	-.593599822-06	-.206037210-06	-.387340208-06
113.45148087	.102059853-06	.298154408-07	.689218327-07
113.45833302	-.593588361-06	-.206294441-06	-.387220862-06
113.45842552	.102057718-06	.298612615-07	.689051687-07
113.46527767	-.593576914-06	-.206551398-06	-.387101416-06
113.46537018	.102055592-06	.299070386-07	.688884683-07
113.47222137	-.593565481-06	-.206808146-06	-.386981831-06
113.47231388	.102053459-06	.299527718-07	.688717412-07

APPENDIX

COMPUTER PROGRAM FOR CALCULATION OF HELIOS MANEUVERS

```
C  THIS PROGRAMME COMPUTES PERTURBATIONS DUE TO ATTITUDE CONTROL MANOEUVRES
C  FOR ARBITRARY NUMBER OF FIRING CYCLES. EACH CYCLE CONSISTS OF 16 PULSES,
C  EACH PULSE LASTING ONE QUARTER OF A SECOND. JET FORCE IS ONE NEWTON.
C
C*******************************************************************************
C
C  NOMENCLATURE-
C
C          AX      = THE SEMI-MAJOR AXIS OF THE SPACECRAFT-S ELLIPTIC
C                      ORBIT (KM)
C          ECC     = THE ORBITAL ECCENTRICITY OF THE SPACECRAFT
C          EPSLN   = OBLIQUITY OF THE ECLIPTIC
C          INCL    = INCLINATION OF THE ORBITAL PLANE TO THE ECLIPTIC
C                      PLANE OF 1950.0
C          NODE    = NODAL ANGLE OF THE ORBITAL PLANE OF SPACECRAFT
C          OMEGA   = ARGUMENT OF THE PERIAPSIS OF THE SPACECRAFT-S ORBIT
C          M       = MEAN ANOMALY OF THE SPACECRAFT
C          E       = ECCENTRIC ANOMALY OF THE SPACECRAFT
C          MSTART  = MEAN ANOMALY OF THE SPACECRAFT AT THE TIME OF
C                      INITIALIZATION   T = TSTART
C          TSTART  = INITIAL TIME
C          GM      = GRAVITATIONAL CONSTANT OF THE SUN
C          MEAN    = MEAN ORBITAL MOTION OF THE SPACECRAFT
C          PER     = ORBITAL PERIOD OF THE SPACECRAFT
C          N       = NUMBER OF POINTS ON THE TRAJECTORY
C          TSTEP   = TIME STEP (DAYS)
C
C     EPOCH = TSTART = 1974, DECEMBER 10, 00 HRS, 00 MIN, 00 SEC.
C
C*******************************************************************************
C
C  SPECIFICATIONS-
C
      REAL MSTART,MZERO,MEAN,LONG ,MASS,INCL,NODE
      DIMENSION X(40),Y(40),Z(40),R(40),TA(40),TIME(40),DAN(40),SAT(40),
     1 EM(40),SEC(40),LONG(40),MANUVR(4),DELX(40),DELY(40),DELZ(40),
     2  DELDX(40),DELDY(40),DELDZ(40),TAU(4),DX(4),DY(4),DZ(4),DDX(4),
     3 DDY(4),DDZ(4),XA(2,40),YA(2,40),ZA(2,40),XACC(80),YACC(80),
     4 ZACC(80),THALF(80)
C
      NAMELIST/INPUT/TSTART,TSTEP,DT,AX,ECC,OMEGA,NODE,INCL,AU,MZERO,
     1 N,TLAUCH,GM,DAY,PI,EPSLN,THETA,MASS,SAT,EM,SEC,ALPHA,DELTA
 7000 READ(5,INPUT)
      WRITE(6,1000)
      WRITE(6,INPUT)
      WRITE(6,3000)
 1000 FORMAT(1H1,1X,/)
 3000 FORMAT(1H1,4X////)
      RAD = 180./PI
C  COMPUTATION OF UNPERTURBED POSITIONS OF THE SPACECRAFT
      EPSLN = EPSLN/RAD
      CE = COS(EPSLN)
      SE = SIN(EPSLN)
      ALPHA = ALPHA/RAD
```

```
      DELTA = DELTA/RAD
      XCAN = COS(ALPHA)*COS(DELTA)
      YCAN = SIN(ALPHA)*COS(DELTA)
      ZCAN = SIN(DELTA)
      MEAN = SQRT(GM/AX**3)
      DM     = MEAN*DAY*RAD
      PER = 2.0*PI/(MEAN*DAY)
      MSTART = MZERO - DM*DT
      IF(MSTART.LT.0.)MSTART=MSTART+360.
 2003 WRITE(6,2)
      WRITE(6,3)AX,ECC,OMEGA,MSTART,DM    ,PER,INCL,NODE
      WRITE(6,32)
    2 FORMAT(36X,-HELIOCENTRIC EQUATORIAL ORBITAL PARAMETERS OF THE -,
     * -SPACECRAFT-//)
    3 FORMAT(38X,-AX =-,E16.8,1X,-KM-/,37X,-ECC =-,F16.11/,35X,-OMEGA =-
     1  ,F16.11,1X,-DEG-/,34X,-MSTART =-,F16.11,1X,-DEG-/,36X,-MEAN =-,
     2   F16.11,1X,-DEG/DAY-/,34X,-PERIOD =-,F16.11,1X,-DAYS-/,36X,
     3  -INCL =-,F16.11,1X,-DEG-/,36X,-NODE =-,F16.11,1X,-DEG-///)
   32 FORMAT(1H1,24X,-TIMES OF FIRING CYCLES-/)
      CALL VECTOR(INCL,NODE,OMEGA,PX,PY,PZ,QX,QY,QZ)
      MSTART = MSTART/RAD
      DO 1 I=1,N
      IF(I.LT.11)DAN(I)=52.
      IF(I.GE.11.AND.I.LT.21)DAN(I)=74.
      IF(I.GE.21.AND.I.LT.28)DAN(I)=91.
      IF(I.GE.28)DAN(I)=113.
      TIME(I) = ( (SEC(I)/60. + EM(I))/60. + SAT(I))/24. + DAN(I)
      WRITE(6,5)TIME(I)
      IF(I.EQ.10.OR .I.EQ.20.OR .I.EQ.27)WRITE(6,6)
    5 FORMAT(25X,F15.10)
    6 FORMAT(1X,/)
    1 CONTINUE
      WRITE(6,33)PX,PY,PZ,QX,QY,QZ
   33 FORMAT(1H1/////,12X,-P =-/,3(15X,F12.7/),//,12X,-Q =-/,3(15X,
     1 F12.7/),//)
      WRITE(6,7)
    7 FORMAT(1H1,/,3X,-EQUATORIAL COORDINATES OF HELIOS-///,4X,-TIME-,
     1 10X,-X(KM)-,12X,-Y(KM)-,12X,-Z(KM)-,12X,-R(KM)-,10X,-TA-,9X,
     2 -LONG-/)
      DO 8 I=1,N
      CALL ORBIT(1,TIME(I),MSTART,AX,ECC,MEAN,PX,PY,PZ,QX,QY,QZ,
     1  X(I),Y(I),Z(I),R(I),TA(I),LONG(I))
      WRITE(6,9)TIME(I),X(I),Y(I),Z(I),R(I),TA(I),LONG(I)
      IF(I.EQ.10.OR .I.EQ.20.OR .I.EQ.27)WRITE(6,91)
    9 FORMAT(F9.2,4E17.7,2F12.3)
   91 FORMAT(2X,/)
    8 CONTINUE
C   CALCULATION OF MANOEUVRES
      THETA = THETA/RAD
      SPIN = 2.*PI
      AL = PI/2.
      BL = (PI**2)/8.
      CL = AL - 1.
      CT = COS(THETA)
      ST = SIN(THETA)
      WRITE(6,98)
   98 FORMAT(1H1,2X/)
      DO 11 I=1,N
      IF(I.LT.11)MANUVR(I) = 1
```

```
      IF(I.GE.11.AND.I.LT.21)MANUVR(2) = 2
      IF(I.GE.21.AND.I.LT.28)MANUVR(3) = 3
      IF(I.GE.28)MANUVR(4) = 4
      SIGN = 1.
      IF(I.GE.3.AND.I.LE.6)SIGN=-1.0
      IF(I.GE.11.AND.I.LE.14)SIGN=-1.0
      IF(I.GE.21.AND.I.LE.27)SIGN=-1.0
      IF(I.GE.35)SIGN=-1.0
      FORCE = SIGN*1.E-03
      WRITE(6,99)FORCE
   99 FORMAT(10X,-FORCE =-,F9.3,1X,-KG.KM/SEC**2-)
      RCOEF = -FORCE/(MASS*SPIN**2)
      VCOEF = -SPIN*RCOEF
      LONG(I) = LONG(I)/RAD
      CX = CL*SIN(LONG(I)) - COS(LONG(I))
      CYZ = CL*COS(LONG(I)) + SIN(LONG(I))
      CVX = -SIN(LONG(I)) + COS(LONG(I))
      CVYZ = -(SIN(LONG(I)) + COS(LONG(I)))
      DRX = RCOEF*CX*ST
      DRY = RCOEF*(CYZ*CE*ST + BL*SE*CT)
      DRZ = RCOEF*(CYZ*SE*ST - BL*CE*CT)
      DVX = VCOEF*CVX*ST
      DVY = VCOEF*(CVYZ*CE*ST - AL*SE*CT)
      DVZ = VCOEF*(CVYZ*SE*ST + AL*CE*CT)
      DELX(I) = 16.*DRX + 132.*DVX
      DELY(I) = 16.*DRY + 132.*DVY
      DELZ(I) = 16.*DRZ + 132.*DVZ
      DELDX(I) = 16.*DVX
      DELDY(I) = 16.*DVY
      DELDZ(I) = 16.*DVZ
   11 CONTINUE
      WRITE(6,15)
   15 FORMAT(1H1,2X,-TIME-,9X,-DX(KM)-,11X,-DY(KM)-,11X,-DZ(KM)-,11X,
     1 -DVX(KM/S)-,8X,-DVY(KM/S)-,8X,-DVZ(KM/S)-/)
      DO 22 I=1,N
      WRITE(6,16)TIME(I),DELX(I),DELY(I),DELZ(I),DELDX(I),DELDY(I),
     *  DELDZ(I)
      IF(I.EQ.10.OR .I.EQ.20.OR .I.EQ.27)WRITE(6,20)
   16 FORMAT(1X,F7.2,6F17.9)
   20 FORMAT(1X,/)
   22 CONTINUE
C********************** ACCUMULATED EFFECTS OF MANOEUVRES ********************
      WRITE(6,4)
    4 FORMAT(1H1,//,10X,-ACCUMULATED DISTURBING EFFECTS IN COORDINATES -
     1  -DURING MANOEUVRES-//)
      DO 10 J=1,4
      DX(J) = 0.0
      DY(J) = 0.0
      DZ(J) = 0.0
      DDX(J) = 0.0
      DDY(J) = 0.0
      DDZ(J) = 0.0
      IF(J.EQ.1)ISTRT=1
      IF(J.EQ.2)ISTRT=11
      IF(J.EQ.3)ISTRT=21
      IF(J.EQ.4)ISTRT=28
      IF(J.EQ.1)NFIN=10
      IF(J.EQ.2)NFIN=20
      IF(J.EQ.3)NFIN=27
```

```
      IF(J.EQ.4)NFIN=38
      TAU(J) = TIME(NFIN)
      DO 12 I=ISTRT,NFIN
      K = I-ISTRT+1
      L = I-1
      DDX(J) = DDX(J) + DELDX(I)
      DDY(J) = DDY(J) + DELDY(I)
      DDZ(J) = DDZ(J) + DELDZ(I)
      IF(K.EQ.1)GO TO 14
      DX(J) = DX(J) + DELX(I)
      DY(J) = DY(J) + DELY(I)
      DZ(J) = DZ(J) + DELZ(I)
      GO TO 12
   14 DELT = (TAU(J) - TIME(L))*DAY
      DX(J) = DX(J) + DELX(I) + DELDX(I)*DELT
      DY(J) = DY(J) + DELY(I) + DELDY(I)*DELT
      DZ(J) = DZ(J) + DELZ(I) + DELDZ(I)*DELT
   12 CONTINUE
      WRITE(6,17)MANUVR(J),DX(J),DY(J),DZ(J),DDX(J),DDY(J),DDZ(J)
   17 FORMAT(7X,-MANOEUVRE NO.-,I5//,14X,-DX =-,F16.8,1X,-KM-/,14X,
     1 -DY =-,F16.8,1X,-KM-/,14X,-DZ =-,F16.8,1X,-KM-/,13X,-DVX =-,
     2 F16.8,1X,-KM/SEC-/,13X,-DVY =-,F16.8,1X,-KM/SEC-/,13X,-DVZ =-,
     3 F16.8,1X,-KM/SEC-///)
   10 CONTINUE
C  CALCULATION OF ACCELERATIONS WHICH SHOULD BE IMPLEMENTED INTO THE ORBIT
C  DETERMINATION PROGRAMME
      DO 18 I=1,N
      A1X      = (DELX(I) - 4.*DELDX(I))/64.
      A1Y      = (DELY(I) - 4.*DELDY(I))/64.
      A1Z      = (DELZ(I) - 4.*DELDZ(I))/64.
      A2X      = (-DELX(I) + 8.*DELDX(I))/32.
      A2Y      = (-DELY(I) + 8.*DELDY(I))/32.
      A2Z      = (-DELZ(I) + 8.*DELDZ(I))/32.
C  TRANSFORMATION INTO THE XSTAR, YSTAR, USP SYSTEM
      UDR = XCAN*X(I) + YCAN*Y(I) + ZCAN*Z(I)
      AMAG = SQRT(R(I)**2 - UDR**2)
      UXX = YCAN*Z(I) - ZCAN*Y(I)
      UXY = ZCAN*X(I) - XCAN*Z(I)
      UXZ = XCAN*Y(I) - YCAN*X(I)
      E1X = (X(I)*UDR - XCAN*R(I)**2)/(AMAG*R(I))
      E1Y = (Y(I)*UDR - YCAN*R(I)**2)/(AMAG*R(I))
      E1Z = (Z(I)*UDR - ZCAN*R(I)**2)/(AMAG*R(I))
      E2X = UXX/AMAG
      E2Y = UXY/AMAG
      E2Z = UXZ/AMAG
      E3X = X(I)/R(I)
      E3Y = Y(I)/R(I)
      E3Z = Z(I)/R(I)
      XA(1,I) = A1X*E1X + A1Y*E1Y + A1Z*E1Z
      YA(1,I) = A1X*E2X + A1Y*E2Y + A1Z*E2Z
      ZA(1,I) = A1X*E3X + A1Y*E3Y + A1Z*E3Z
      XA(2,I) = A2X*E1X + A2Y*E1Y + A2Z*E1Z
      YA(2,I) = A2X*E2X + A2Y*E2Y + A2Z*E2Z
      ZA(2,I) = A2X*E3X + A2Y*E3Y + A2Z*E3Z
      I1 = 2*I-1
      I2 = 2*I
      XACC(I1) = XA(1,I)
      XACC(I2) = XA(2,I)
      YACC(I1) = YA(1,I)
```

```
      YACC(I2) = YA(2,I)
      ZACC(I1) = ZA(1,I)
      ZACC(I2) = ZA(2,I)
      THALF(I1) = TIME(I)
      THALF(I2) = TIME(I) + 8./DAY
   18 CONTINUE
      NT2 = 2*N
      DO 21 I=1,NT2
      IF(I.EQ.1.OR.I.EQ.41)WRITE(6,19)
      WRITE(6,23)THALF(I),XACC(I),YACC(I),ZACC(I)
      IF(I.EQ.20.OR .I.EQ.40.OR .I.EQ.54)WRITE(6,24)
   19 FORMAT(1H1,//,3X,-DISTURBING ACCELERATIONS IN THE XSTAR, YSTAR, -
     1 -USP SYSTEM IN KM/SEC**2-//,3X,-TIME-,15X,-XACC-,13X,-YACC-,13X,
     2 -ZACC-/)
   23 FORMAT(1X,F13.8,3E17.9)
   24 FORMAT(1X,/)
   21 CONTINUE
      WRITE(6,2000)
 2000 FORMAT(1H1,2X/)
      STOP
      END
```

```
      SUBROUTINE VECTOR(INCL,NODE,OMEGA,PX,PY,PZ,QX,QY,QZ)
C   THIS SUBROUTINE COMPUTES THE COMPONENTS OF VECTORS P, Q, AND R.
      REAL INCL,NODE
      DATA PI/3.141592654/
      RAD = 180./PI
      INCL = INCL/RAD
      NODE = NODE/RAD
      OMEGA = OMEGA/RAD
      CI = COS(INCL)
      SI = SIN(INCL)
      CN = COS(NODE)
      SN = SIN(NODE)
      CO = COS(OMEGA)
      SO = SIN(OMEGA)
      PX = CN*CO - SN*SO*CI
      PY = SN*CO + CN*SO*CI
      PZ = SO*SI
      QX = -CN*SO - SN*CO*CI
      QY = -SN*SO + CN*CO*CI
      QZ = CO*SI
      RX = SN*SI
      RY = -CN*SI
      RZ = CI
      RETURN
      END
```

```
      SUBROUTINE ORBIT(BODY,TIME,MSTART,SMA,ECC,MEAN,PX,PY,PZ,QX,
     1  QY,QZ,X,Y,Z,R,T,LONG)
C  THIS SUBROUTINE COMPUTES HELIOCENTRIC EQUATORIAL COORDINATES OF HELIOS
      INTEGER BODY
      REAL M,MSTART,MEAN,LONG
      DATA T0/5.0/,PI/3.141592654/,DAY/86400./,EPSLN/23.445789/
      RAD = 180./PI
      EPSLN = EPSLN/RAD
      ETA = SQRT((1.0+ECC)/(1.0-ECC))
      M = MSTART + (TIME-T0)*DAY*MEAN
      E = ANOM(ECC,M)
      R = SMA*(1.-ECC*COS(E))
      T = 2.*ATAN(ETA*TAN(E/2.))
      IF(T.GE.2.*PI)T=T-2.*PI
      XORB = R*COS(T)
      YORB = R*SIN(T)
      T = T*RAD
      IF(T.LT.0.)T=T+360.
      X = PX*XORB + QX*YORB
      Y = PY*XORB + QY*YORB
      Z = PZ*XORB + QZ*YORB
      SE = SIN(EPSLN)
      CE = COS(EPSLN)
C  ECLIPTIC LONGITUDE OF HELIOS
      CL = X/R
      SL = (Y*CE + Z*SE)/R
      LONG = ATAN2(SL,CL)
      LONG = LONG*RAD
      IF(LONG.LT.0.)LONG=LONG+360.
      RETURN
      END
```

```
      FUNCTION ANOM(ECC,M)
C   THIS FUNCTION SUBROUTINE SOLVES THE KEPLER-S EQUATION BY ITERATIONS
      REAL M
      DATA EPS/.000005/
      ANOM = M
    2 ANOM = M + ECC*SIN(ANOM)
      TEST = ANOM - M - ECC*SIN(ANOM)
      IF(ABS(TEST).GT.EPS)GO TO 2
      IF(ABS(TEST).LE.EPS)RETURN
      END
```

```
$INPUT
ECC = .521807390542
TSTART = 0.0
TSTEP = 86400.0
AX = .96001973563E+08
ALPHA = 98.02255
DELTA = -68.98877
EPSLN = 23.44578889
OMEGA = 257.444908784
INCL = 23.4469891399
NODE = .0613804058997
MZERO = 71.6320412103
TLNCH = -5.
AU = .1495978930E+9
GM = .132712499390802500E+12
MASS = 356.9
DT = 128.2152777778
DAY = 86400.
N = 38
PI = 3.141592654
THETA = 38.
SAT = 2*7.,4*12.,13.,2*14.,15.,7.,3*8.,9.,10.,14.,2*15.,16.,2*7.,2*8.,
  6*9.,5*10.,3*11.
EM = 29.,38.,6.,15.,24.,33.,56.,24.,52.,20.,55.,9.,22.,35.,36.,6.,12.,
  17.,49.,15.,29.,50.,30.,50.,10.,30.,50.,9.,40.,50.,0.,10.,20.,30.,
  50.,0.,10.,20.
SEC = 11*0.,21.,22.,43.,59.,44.,45.,39.,51.,53.,7*0.,13.,10*0.
$END
-FIN                                                                  NIF-
```

References

1. Georgevic, R. M., Mction of the Sun-Canopus Oriented Attitude Control Reference Frame of the Mariner Venus/Mercury Spacecraft, Technical Memorandum 391-429, Jet Propulsion Laboratory, Pasadena, Calif., March 30, 1973 (an internal document).

Bibliography

1. Georgevic, R. M., Rigid Body Dynamics, Lecture Notes, 1967-1968.

NASA – JPL – Coml. L A . Calif